1 An experiment consists of rolling a pair of dice and observing the uppermost faces. The sample space for this experiment consists of 36 outcomes listed as pairs of numbers:

$$S = \{(1,1), (1,2), \cdots, (6,6)\}$$

Let E be the event that both faces are even and F the event that both faces add to 6. Which of the following statements is true?

(a) E and F are mutually exclusive (b) $E \cap F = \{(4, 2), (2, 4)\}$ (c) $E \cup F = S$ (d) $E \cup F' = S$ (e) $E \cap F' = \emptyset$

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 - Hence (b) is true.

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 - Alternatively we could write out the entire sample space in this case and identify the events E and F :

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1,5)	(1, 6)
(2, 1)	(2,2)	(2,3)	(2, 4)	(2,5)	(2, 6)
(3, 1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4, 1)	(4, 2)	(4,3)	(4, 4)	(4,5)	(4,6)
(5, 1)	(5,3)	(5,3)	(5,4)	(5,5)	(5,6)
(6, 1)	(6,2)	(6,3)	(6,4)	(6,5)	(6, 6)

Here the event $F = {Faces sum to 6}$ is shown in Green.

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Here the event F = {Faces sum to 6} is shown in Green.	(1, 6)(2, 6)(3, 6)(4, 6)(5, 6)(6, 6)	(1,5)(2,5)(3,5)(4,5)(5,5)(6,5)	(1, 4)(2, 4)(3, 4)(4, 4)(5, 4)(6, 4)	(1,3)(2,3)(3,3)(4,3)(5,3)(6,3)	(1, 2)(2, 2)(3, 2)(4, 2)(5, 3)(6, 2)	(1,1)(2,1)(3,1)(4,1)(5,1)(6,1)
Here the event E = {Both faces are even} is shown in Red.	(1, 6)(2, 6)(3, 6)(4, 6)(5, 6)(6, 6)	(1,5) (2,5) (3,5) (4,5) (5,5) (6,5)	(1, 4)(2, 4)(3, 4)(4, 4)(5, 4)(6, 4)	(1,3)(2,3)(3,3)(4,3)(5,3)(6,3)	(1,2)(2,2)(3,2)(4,2)(5,3)(6,2)	(1,1)(2,1)(3,1)(4,1)(5,1)(6,1)

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$\begin{array}{cccccccc} (1,1) & (1) \\ (2,1) & (2) \\ (3,1) & (3) \\ (4,1) & (4) \\ (5,1) & (5) \\ (6,1) & (6) \end{array}$	1,2) (2,2) (3,2) (4,2) (5,3) (5,2) ((1,3) (2,3) (3,3) (4,3) (5,3) (6,3)	(1, 4) (2, 4) (3, 4) (4, 4) (5, 4) (6, 4)	(1,5)(2,5)(3,5)(4,5)(5,5)(6,5)	(1,6) (2,6) (3,6) (4,6) (5,6) (6,6)	Here the event F = {Faces sum to 6} is shown in Green.
$\begin{array}{c} (1,1) & (1) \\ (2,1) & (2) \\ (3,1) & (3) \\ (4,1) & (4) \\ (5,1) & (5) \\ (6,1) & (6) \end{array}$	1,2) (2,2) (3,2) (4,2) (5,3) (5,2) ((1,3)(2,3)(3,3)(4,3)(5,3)(6,3)	(1, 4)(2, 4)(3, 4)(4, 4)(5, 4)(6, 4)	(1, 5) (2, 5) (3, 5) (4, 5) (5, 5) (6, 5)	(1, 6)(2, 6)(3, 6)(4, 6)(5, 6)(6, 6)	Here the event E = {Both faces are even} is shown in Red.

▶ By comparing both pictures we can see that (b) is the only true answer.

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2 Let *E* and *F* be events where $Pr(E') = \frac{1}{4}$, $Pr(F) = \frac{1}{4}$, and $Pr(E \cap F) = \frac{1}{8}$. Find $Pr(E \cup F)$. (a) $\frac{7}{8}$ (b) $\frac{5}{8}$ (c) $\frac{3}{8}$ (d) $\frac{1}{2}$ (e) 1

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The correct answer is (a).

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▶ The answer is (d).

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4 A box, ready for shipment, contains 20 lightbulbs, 4 of which are defective. An inspector selects a sample of 3 lightbulbs from the box. If the inspector finds at least one defective lightbulb among those sampled, the box will not be shipped, otherwise the box will be shipped. What is the probability that this box will pass the inspection and be shipped?

(a) $\frac{C(20,3)}{P(20,3)}$ (b) $\frac{C(4,3) \cdot C(16,3)}{C(20,3)}$ (c) $\frac{C(16,3)}{C(20,3)}$ (d) $\frac{C(4,3)}{C(20,3)}$ (e) $1 - \frac{C(4,3)}{C(20,3)}$.

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We have a box, with 4 defective lightbulbs and 16 good ones, from which a sample of size 3 will be drawn at random 4D 16G

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5



The above is a map of the roads in a country town. A motorist travels from A to C (traveling East or South only). If the motorist is equally likely to choose any of the routes from A to C, what is the probability that their car will pass through B? (a) $\frac{1}{7}$ (b) $\frac{11}{63}$ (c) $\frac{4}{21}$ (d) $\frac{13}{63}$ (e) $\frac{10}{63}$

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Since all routes are equally likely to be chosen, the probability that the car pass through the point B is:

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$$= \frac{C(4,1)C(5,4)}{C(9,5)} = \frac{4\cdot5}{126} = \frac{20}{126} = \frac{10}{63}$$

The correct answer is (e)

6 In tossing a fair die, we observe the uppermost face. Let *E* be the event "an odd number occurs" and let F be the event "a number greater than 3 occurrs". What is P(E|F)?.

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$$P(E \cap F) = 1/6 \text{ and } P(F) = 3/6$$

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$$E \cap F = \{1, 3, 5\} \cap \{4, 5, 6\} = \{5\}.$$

• Therefore
$$P(E|F) = \frac{1/6}{3/6} = 1/3$$

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► The correct answer is (c)

7 A new piece of electronic equipment has five components. the probability of failure within a year is 0.1 for each component. Assuming that the failure of the various components are independent of each other, what is the probability that no component will fail in the first year?

(a) $(0.1)^5$ (b) $1 - (0.1)^5$ (c) $1 - (0.9)^5$ (d) $(0.9)^5$ (e) 0.1

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For each component, the probability that it does not fail in the first year is 1 - Pr(fails in first year) = 1 - (0.1) = 0.9, by the complement rule.

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= Pr(1st does not fail and 2nd does not fail and 3rd does not fail and 4th does not fail and 5th does not fail)

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= Pr(1st does not fail and 2nd does not fail and 3rd does not fail and 4th does not fail and 5th does not fail)

= $Pr(1st \text{ does not fail}) \cdot Pr(2nd \text{ does not fail}) \cdot Pr(3rd \text{ does not fail}) \cdot Pr(4th \text{ does not fail}) \cdot Pr(5th \text{ does not fail}).$

7 A new piece of electronic equipment has five components. the probability of failure within a year is 0.1 for each component. Assuming that the failure of the various components are independent of each other, what is the probability that no component will fail in the first year?

(a) $(0.1)^5$ (b) $1 - (0.1)^5$ (c) $1 - (0.9)^5$ (d) $(0.9)^5$ (e) 0.1

- For each component, the probability that it does not fail in the first year is 1 Pr(fails in first year) = 1 (0.1) = 0.9, by the complement rule.
- Since the failures of the components are independent of each other, the probability that none fail within a year

= Pr(1st does not fail and 2nd does not fail and 3rd does not fail and 4th does not fail and 5th does not fail)

= $Pr(1st \text{ does not fail}) \cdot Pr(2nd \text{ does not fail}) \cdot Pr(3rd \text{ does not fail}) \cdot Pr(4th \text{ does not fail}) \cdot Pr(5th \text{ does not fail}).$

- ► = (0.9)⁵
- The correct answer is (d)

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8 A magician's hat contains 3 rabbits, a squirrel and a groundhog. The magician pulls animals out of her hat at random, stopping when she runs out of animals. What is the probability that the third animal she pulls out of her hat is a groundhog?

(a) $\frac{1}{10}$ (b) $\frac{1}{5}$ (c) $\frac{3}{10}$ (d) $\frac{3}{20}$ (e) $\frac{1}{4}$

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We can make a tree diagram to represent the outcomes for this experiment:



8 A magician's hat contains 3 rabbits, a squirrel and a groundhog. The magician pulls animals out of her hat at random, stopping when she runs out of animals. What is the probability that the third animal she pulls out of her hat is a groundhog? (a) $\frac{1}{10}$ (b) $\frac{1}{5}$ (c) $\frac{3}{10}$ (d) $\frac{3}{20}$ (e) $\frac{1}{4}$

The outcomes where the groundhog is the third animal pulled out are those paths ending in a G, those marked in red below:



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The outcomes where the groundhog is the third animal pulled out are those paths ending in a G, those marked in red below:



▶ The probability that the third animal pulled out of the hat is a groundhog is the sum of the probabilities of the three outcomes RRG, RSG, and SRG. = $\frac{3}{5} \cdot \frac{1}{2} \cdot \frac{1}{3} + \frac{3}{5} \cdot \frac{1}{4} \cdot \frac{1}{3} + \frac{1}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} = \frac{6+3+3}{60} = \frac{12}{60} = \frac{1}{5}$

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The outcomes where the groundhog is the third animal pulled out are those paths ending in a G, those marked in red below:



• The probability that the third animal pulled out of the hat is a groundhog is the sum of the probabilities of the three outcomes RRG, RSG, and SRG. $= \frac{3}{5} \cdot \frac{1}{2} \cdot \frac{1}{3} + \frac{3}{5} \cdot \frac{1}{4} \cdot \frac{1}{3} + \frac{1}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} = \frac{6+3+3}{60} = \frac{12}{60} = \frac{1}{5}$

The correct answer is (b)

 ${\bf 9}$ The following is a histogram for the probability distribution of a random variable X.



What is $P(X \le 3)$? (a) 0.4 (b) 0.6 (c) 0.8 (d) 0.2 (e) 0.5

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 ${\bf 9}$ The following is a histogram for the probability distribution of a random variable X.



What is $P(X \le 3)$? (a) 0.4 (b) 0.6 (c) 0.8 (d) 0.2 (e) 0.5 $P(X \le 3) = P(X = 1) + P(X = 2) + P(X = 3) = 0.1 + 0.2 + 0.2 = 0.5$

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 ${\bf 9}$ $\;$ The following is a histogram for the probability distribution of a random variable X.



What is $P(X \le 3)$? (a) 0.4 (b) 0.6 (c) 0.8 (d) 0.2 (e) 0.5 $P(X \le 3) - P(X - 1) + P(X - 2) + P(X - 3) - 0.1 + 0.2 + 0.2 - 0.5$

•
$$P(X \le 3) = P(X = 1) + P(X = 2) + P(X = 3) = 0.1 + 0.2 + 0.2 = 0.5$$

► The correct answer is (e)

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(a) How many outcomes are in the sample space for this experiment?

(b) Let E be the event that you observe a 6 on the twelve sided die, what is the probability of E?

(c) Let F be the event that the sum of the numbers on the uppermost faces is 6. What is the probability of F?

(d) Are E and F Mutually Exclusive?

(e) Are *E* and *F* Independent?

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(a) How many outcomes are in the sample space for this experiment?

• There are $6 \times 12 = 72$ pairs of numbers in the sample space.

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► $E = \{(1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)\}$. Because all outcomes in the sample space are equally likely, $Pr(E) = \frac{\#E}{\#S.S.} = \frac{6}{72}$

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 - Yes because $E \cap F = \emptyset$
- (e) Are *E* and *F* Independent?
 - No Because $Pr(E) \cdot Pr(F) = \frac{6}{72} \frac{5}{72} \neq Pr(E \cap F)$

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(a) How many lists of five numbers can be made using the numbers 1-20 (inclusive), if repetitions are allowed?

(b) How many lists of five numbers can be made using the numbers 1-20 (inclusive), if repetitions are not allowed?

(c) If five people each choose a number at random (and independently) from the numbers 1-20 (inclusive), what is the probability that all five numbers will be different?

(d) If five people each choose a number at random (and independently) from the numbers 1-20 (inclusive), what is the probability that at least two of the numbers will be the same?

(a) How many lists of five numbers can be made using the numbers 1-20 (inclusive), if repetitions are allowed?

• We have $\underline{20} \cdot \underline{20} \cdot \underline{20} \cdot \underline{20} \cdot \underline{20} = 20^5$ such lists.

(b) How many lists of five numbers can be made using the numbers 1-20 (inclusive), if repetitions are not allowed?

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(b) How many lists of five numbers can be made using the numbers 1-20 (inclusive), if repetitions are not allowed?

• We have $\underline{20} \cdot \underline{19} \cdot \underline{18} \cdot \underline{17} \cdot \underline{16} = P(20,5)$ such lists.

(c) If five people each choose a number at random (and independently) from the numbers 1-20 (inclusive), what is the probability that all five numbers will be different?

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(c) If five people each choose a number at random (and independently) from the numbers 1-20 (inclusive), what is the probability that all five numbers will be different?

 Because the numbers are chosen randomly, all 20⁵ lists from part (a) are equally likely to occur. Therefore Pr(all five numbers are different) = <u>#lists with 5 different numbers</u> = P(20,5) <u>Total number of lists</u> = P(20,5) <u>20⁵</u>.

(d) If five people each choose a number at random (and independently) from the numbers 1-20 (inclusive), what is the probability that at least two of the numbers will be the same?

(a) How many lists of five numbers can be made using the numbers 1-20 (inclusive), if repetitions are allowed?

• We have $\underline{20} \cdot \underline{20} \cdot \underline{20} \cdot \underline{20} \cdot \underline{20} = 20^5$ such lists.

(b) How many lists of five numbers can be made using the numbers 1-20 (inclusive), if repetitions are not allowed?

• We have $\underline{20} \cdot \underline{19} \cdot \underline{18} \cdot \underline{17} \cdot \underline{16} = P(20,5)$ such lists.

(c) If five people each choose a number at random (and independently) from the numbers 1-20 (inclusive), what is the probability that all five numbers will be different?

• Because the numbers are chosen randomly, all 20^5 lists from part (a) are equally likely to occur. Therefore $Pr(all \text{ five numbers are different}) = \frac{\#\text{lists with 5 different numbers}}{\text{Total number of lists}} = \frac{P(20,5)}{20^5}$.

(d) If five people each choose a number at random (and independently) from the numbers 1-20 (inclusive), what is the probability that at least two of the numbers will be the same?

► By the complement rule $Pr(at \text{ least two will be the same}) = 1 - Pr(all five numbers are different) = 1 - <math>\frac{P(20,5)}{20^5}$.

 $13(10\ \text{Pts.})$ A group of 400 students at a small college were studied and information regarding gender and color blindness status was collected from each individual. The results of the study are recorded in the following table:

Color Blindness
status

	Male	Female
Yes	10	4
No	190	196

Let C denote the event that an individual selected at random from the group is color blind and let M denote the event that an individual selected at random from the group is male.

(a) What is the probability that a randomly selected individual from the group is color blind? that is, what is P(C)?

(b) Given that a male is selected at random from this group, what is the probability that he is color blind? that is, what is P(C|M)?

(c) Are the events C and M independent? Give a reason for your answer.

(d) Are the events C and M Mutually Exclusive? Give a reason for your answer.

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Let C denote the event that an individual selected at random from the group is color blind and let M denote the event that an individual selected at random from the group is male.

(a) What is the probability that a randomly selected individual from the group is color blind? that is, what is P(C) ? P(C) = 14/400.

(b) Given that a male is selected at random from this group, what is the probability that he is color blind? that is, what is P(C|M)?

(c) Are the events C and M independent? Give a reason for your answer.
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 $P(C|M) = \frac{P(C \cap M)}{P(M)} = \frac{10/400}{200/400} = 10/200$

(c) Are the events C and M independent? Give a reason for your answer.

(d) Are the events C and M Mutually Exclusive? Give a reason for your answer.

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(b) Given that a male is selected at random from this group, what is the probability that he is color blind? that is, what is P(C|M)?

 $P(C|M) = \frac{P(C \cap M)}{P(M)} = \frac{10/400}{200/400} = 10/200$

(c) Are the events C and M independent? Give a reason for your answer. *C* and *M* are not independent because $P(C) = 7/200 \neq P(C|M) = 10/200$. (d) Are the events C and M Mutually Exclusive? Give a reason for your answer.

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probability that he is color blind? that is, what is P(C|M) ?

 $P(C|M) = \frac{P(C \cap M)}{P(M)} = \frac{10/400}{200/400} = 10/200$

(c) Are the events C and M independent? Give a reason for your answer. C and M are not independent because $P(C) = 7/200 \neq P(C|M) = 10/200$. (d) Are the events C and M Mutually Exclusive? Give a reason for your answer. C and M are not mutually exclusive, because $C \cap M$ is not empty. there are 10 students in $C \cap M$. < ≣ >

15(10 points) The rules of a carnival game are as follows:

- You pay \$1 to play the game.
- The game attendant then flips a coin at most 4 times.
- As soon as the game attendant gets 2 heads or 3 tails, he stops flipping the coin.
- If the game attendant gets 2 heads, he gives you \$2 (you win).
- ▶ If the game attendant gets 3 tails, he gives you nothing (you lose).
- (a) Draw a tree diagram representing the possible outcomes of the game.
- (b) What is the probability that you win?
- (c) Let X denote the earnings for this game. What are the possible values for X?
- (d) Give the probability distribution of X.

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- (a) Draw a tree diagram representing the possible outcomes of the game.



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- ▶ If the game attendant gets 3 tails, he gives you nothing (you lose).
- (a) Draw a tree diagram representing the possible outcomes of the game.



(b) What is the probability that you win? The winning paths are shown in red above. $P(Win) = (0.5)^2 + 2(0.5)^3 + 3(0.5)^4 = 0.6875$

(c) Let X denote the earnings for this game. What are the possible values for X?

- You pay \$1 to play the game.
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(c) Let X denote the earnings for this game. What are the possible values for X? -1 (lose), 1 (win)

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- (a) Draw a tree diagram representing the possible outcomes of the game.



(b) What is the probability that you win? The winning paths are shown in red above. $P(Win) = (0.5)^2 + 2(0.5)^3 + 3(0.5)^4 = 0.6875$

(c) Let X denote the earnings for this game. What are the possible values for X? -\$1 (lose), \$1 (win)

k	Pr(X = k)
-\$1 (lose)	1 - 0.6875 = 0.3125
\$1 (win)	0.6875

14(10 pts) In a survey conducted on campus, 20 students were asked how many times they had checked their e-mail on the previous day. The results were as follows:

- (a) Organize the data in the relative frequency table below:

Outcome	Frequency	Relative Frequency

(b) Draw a histogram for the data on the axes provided below:

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- (a) Organize the data in the relative frequency table below:

Outcome	Frequency	Relative
		Frequency
1	1	1/20
2	7	7/20
3	6	6/20
4	4	4/20
5	1	1/20
10	1	1/20

(b) Draw a histogram for the data on the axes provided below:

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4	4	4/20
5	1	1/20
10	1	1/20

- (b) Draw a histogram for the data on the axes provided below:
 - A histogram will be drawn on the blackboard

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